Connecting Industry to Mathematics Instruction

NSF ATE Award # 1954291

Solutions: Moving Dirt

Student Activity Sheet

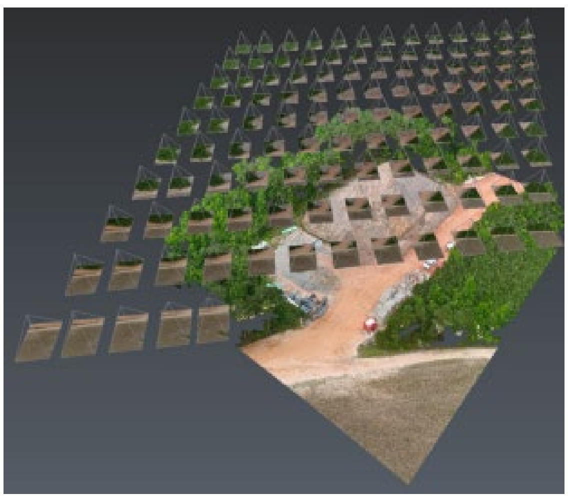
Goal: Find the volume of a large pile of dirt

In partnership with

WAKE TECH

WAKE COUNTY PUBLIC SCHOOL SYSTEM

WakeEd PARTNERSHIP



**Vocabulary and Formulas provided at the end of the document

1. Imagine that you have a rock. What is its volume?
Estimate its volume within 10 seconds of first seeing it.
List ways to estimate the volume of this rock that may take more than 10 seconds. Be prepared to discuss the accuracy of your method when compared to others' methods.

Suggestions for engaging class - go outside a pick up actual rocks. Have rulers, measuring beakers, water available.

Possible answers:
Students may measure and treat rock like a prism or cylinder.
Students may find a volume of water in a beaker with no rock, add rock, find new volume, and subtract.

2. What if instead of a rock, you had a large pile of dirt? Would your ideas above help you find the volume of dirt? Explain.

Answers will vary - students will likely attempt to make the pile into a familiar shape to measure volume (prism, pyramid, cylinder).

possible answer based on diagram: 1,512 ft^3
(21 ft x 18 ft x 4 ft)

What if your large pile of dirt is as large as a building or a mountain?

Surveyors use a tool called a total station to measure the dimensions around the plot of interest (in this case, the sides of the pile).

Figure 3: Total station operation (left), Back-site (right image shown on a yellow tripod), GPS shown on right image as black pole with tablet and round antenna

You are tasked with using a closed traverse and a total station to measure the dimensions of the pile of dirt (shown above and again in figure 2). You walk the traverse and collect data (as shown in figure 1 and 1b below)

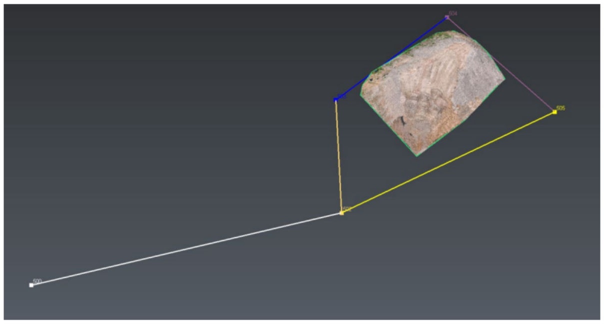


Figure 2: Survey Traverse Path around a Stockpile. The two known points are at the end of the white line, with the three new locations being "turned in" by the total station

Point Number	Description	Horizontal Angle (Decimal Degrees)	Zenith Angle (Decimal Degrees)	Slope Distance
3	NAIL	102.9669444	91.85111111	160.075
4	NAIL	230.5805556	90.21305556	236.691
5	NAIL	275.1488889	88.34666667	208.37
1002	TIE NAIL	279.5558333	90.01722222	330.44
1001	TIE NAIL	191.7463889	89.5675	360.7

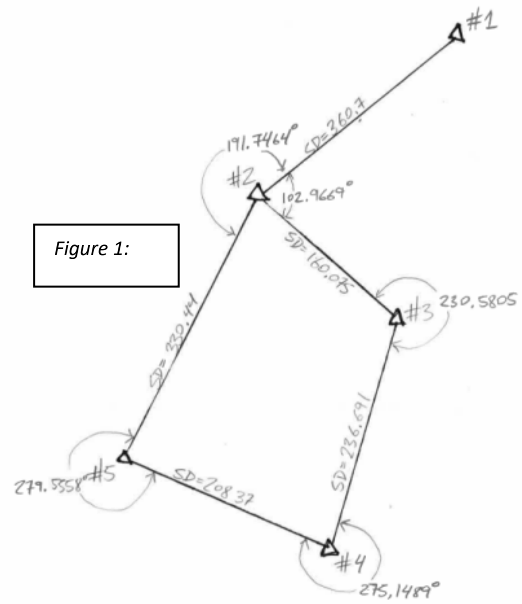
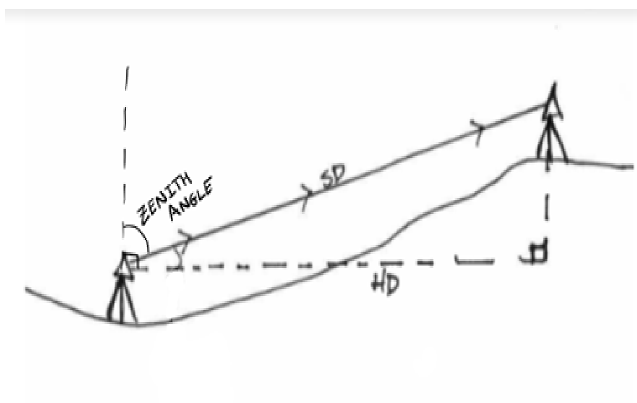


Figure 1:

Figure 1b

When total stations are set up, the ground is often unlevel (we could be surveying on a hilly plot or a steep mountain). Our total station returns a Slope Distance (SD). We can use the SD and our knowledge of SOH-CAH-TOA to find the Horizontal Distance (HD) given the zenith angle.

3. Look at the diagram at right showing a total station "hitting" a set target.



a. If we are interested in finding the dimensions of the foundation of a large building, which value would we likely use, SD or HD? Explain.

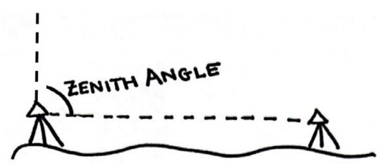
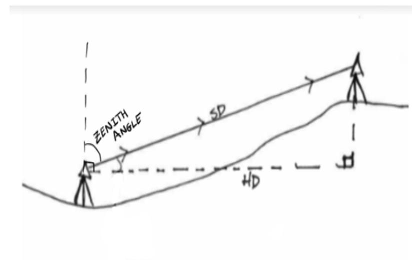
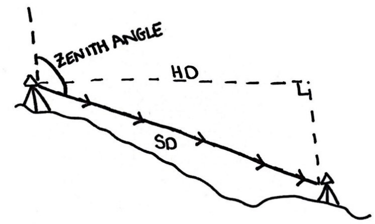
HD (aka Horizontal distance) would represent the length of the base of our building)

b. If we are interested in finding the length of a road going up the mountain or hill, which value would we likely use, SD or HD? Explain.

SD (aka Slope distance) would represent the length of the road)

4. **Zenith angles can measure between 0 and 180 degrees. Match the following diagrams with their zenith angle values. Describe the relationships between the total stations in each situation.** ***Zenith angle choices: 70 degrees, 92 degrees, 115 degrees***

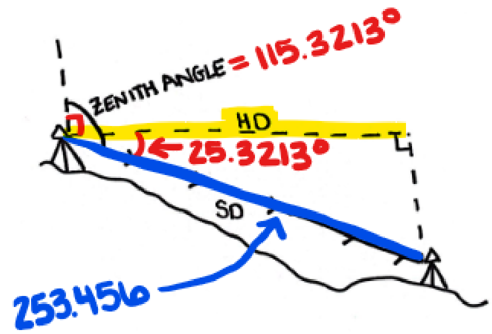
****check out this awesome (and short) video about zenith angles**

scenario 1: 92°	scenario 2: 70°	scenario 3:115°
		

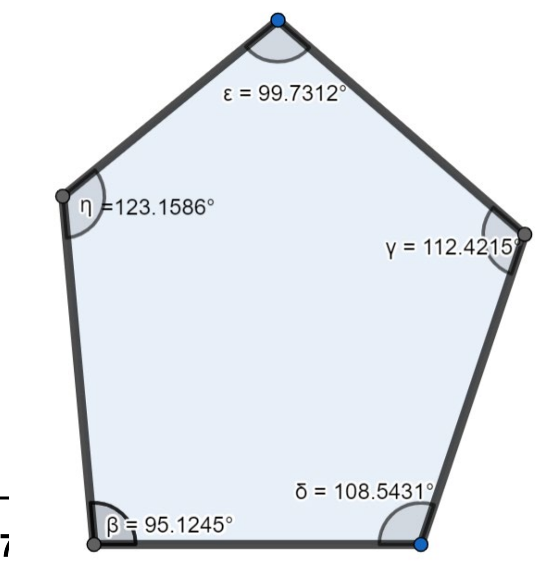
5. **On your Traverse, you set up the Total Station on a point and shot the next point at a slope distance of 253.456 ft. Your Zenith Angle for the shot was 115.3213°. What is the Horizontal distance of this shot?**

cos(25.3213)=HD/253.456

HD = 229.105 feet



6. **Sketch a diagram of the 5 sided traverse loop described below.**



Point Number	Interior angle
101	95.1245°
102	108.5431°
103	112.4215°
104	99.7312°
105	123.1586°

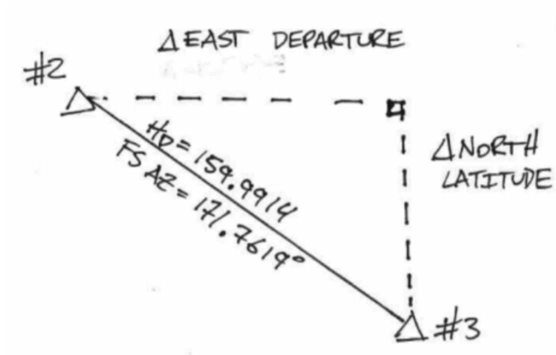
7 ... tely perfect, what is the expected sum of the i... oop? (5 - 2) (180) = 540 degrees

8. **Calculate the angular misclosure of the traverse loop described in question 6. Why might this misclosure exist? 1.0211degrees**

possible answers: human error, rounding error, error in tools used

Traverses have sides that are not horizontal (east-west) and vertical (north-south). We can use information from our total station to find the horizontal and vertical components of the sides of the traverses.

Horizontal components are referred to as Departures and Vertical components are referred to as Latitudes.



- ***Latitude= (Horizontal Distance)* cos(Azimuth)***
- ***Departure= (Horizontal Distance)* sin(Azimuth)***

To find these components, we need to convert Horizontal Angles to Azimuths (see appendix in the back for an explanation of these calculations). We are given the following conversions of the following Horizontal Angles from our 5 sided traverse loop to Azimuths given a starting Backsight Azimuth of 88.1242°.

Backsight Azimuth	Point Number	Horizontal Angle	Azimuth	New Backsight Azimuth
88.1242	101	95.1245	183.2487	3.2487
3.2487	102	108.5431	111.7918	291.7918
291.7918	103	112.4215	44.2133	224.2133
224.2133	104	99.7312	323.9445	143.9445
143.9445	105	123.1586	267.1031	

9. In a traverse, when shooting from point #2 to point #3, your Horizontal Distance is 253.3886 ft and you have a foresight azimuth angle of 111.7918°. Calculate the Latitude and Departure values between the points.

*****Latitude= (Horizontal Distance)* cos(Azimuth)***

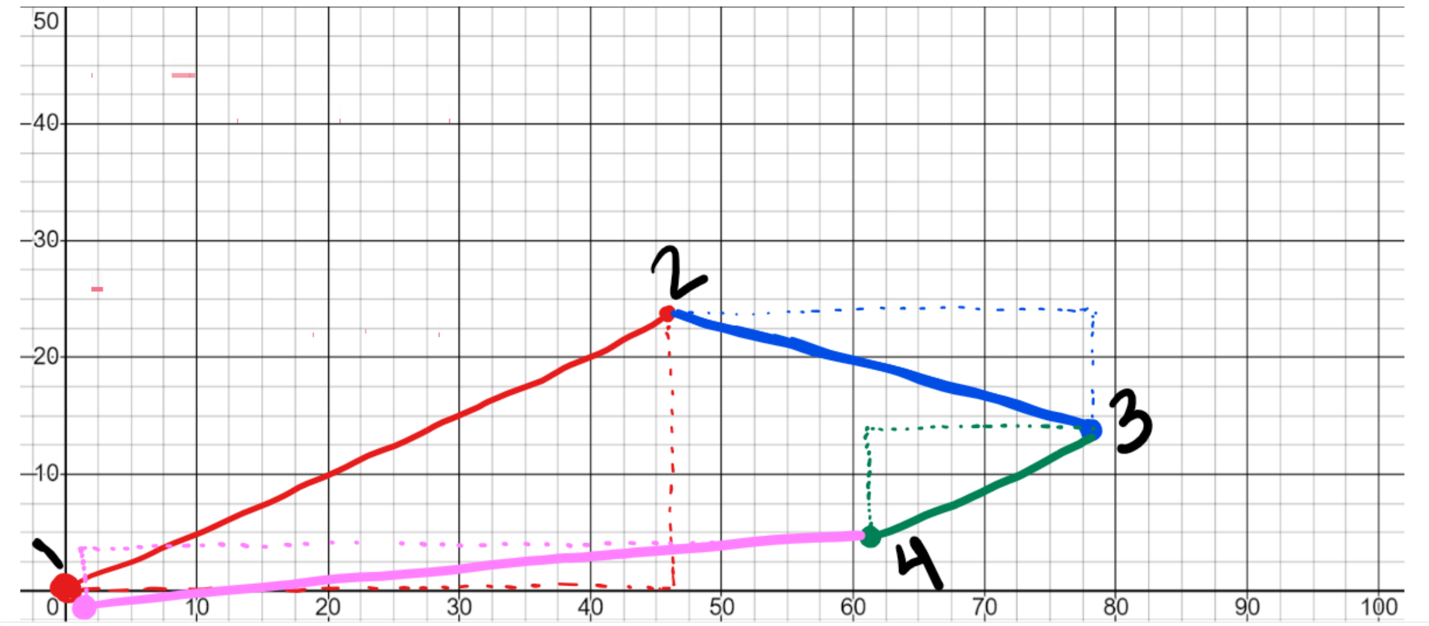
Latitude = (253.3886) * (cos 111.7918) = -94.067 ft

- ***Departure= (Horizontal Distance)* sin(Azimuth)***

Departure = (253.3886) * (sin 111.7918) = 235.281 ft

10. With the following Latitude and Departure values for a four sided closed traverse, sketch the 4 sided loop. Calculate the linear misclosure of the 4 sided loop.

Point Number	Latitude	Departure
101	46.1264	23.3251
102	32.4542	-10.2399
103	-17.2154	-7.2134
104	-60.9726	-5.6026
Σ	0.3926	0.2692



$$\text{Linear Misclosure} = \sqrt{\left(\sum \text{latitudes}\right)^2 + \left(\sum \text{departures}\right)^2}$$

=0.3102

(equations are at the end of the packet)

11. If the sum of the length traversed in problem #11 is 165.1292, what is the ratio of precision for this traverse loop? What does this mean?

Ratio of precision = 532.331 meaning that if we traversed 531.331 feet, we'd have 1 ft of error.

12. What have you created when you find a closed traverse that is accurate to your pile? What additional information is needed to find the volume of the pile?

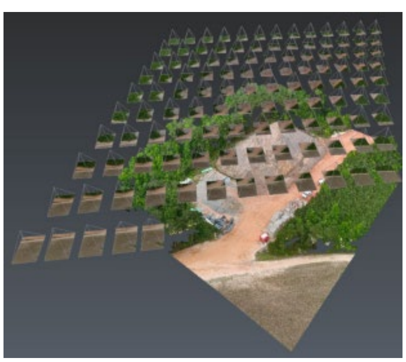
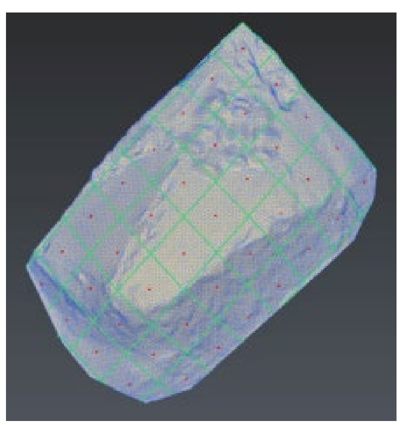
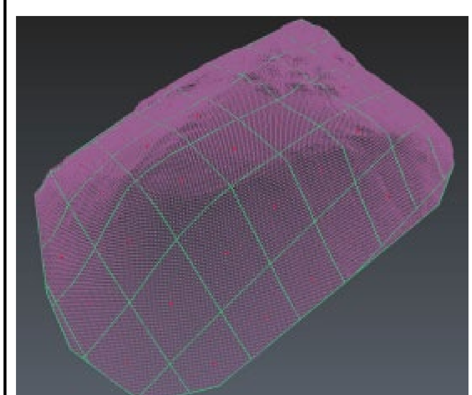
We've effectively created an outline of the base. Now we need the height of the dirt.

Part 2: Stockpile Volume Calculations

The previous example utilized a total station to establish control points around the base of the stockpile. That same device can be utilized to gather spot locations across the stockpile. This conventional survey method can take a good amount of time to walk across the stockpile and can also present some safety issues if the pile is very steep or has active construction equipment working on or around it.

As technology has improved, we are able to harness the capability of Unmanned Aircraft Systems (UAS) to generate 3D topographic survey information from “old school” photogrammetry principles. When a single point is photographed from multiple angles it can be digitally reconstructed so that its location is represented by 3D points that have an X, Y, and Z location. This methodology can also be viewed as “virtual surveying” since the volumetrics are derived from computer software, but the mathematics are still the same as if they were derived from boots on the ground survey locations.

Figure 4: Fixed Wing UAS/Drone (Wingtra) Setup	Figure 5: Photogrammetry Flight
	

Figure 7: SfM 3D Point Cloud & Image Locations	Figure 8: Virtual Survey 25ft Fishnet Grid (Red Dots)	Figure 9: Virtual Survey 1ft Fishnet Grid (Purple)
		

We will use 2 methods to estimate volume:

I. Accumulate values determined when multiplying area of a 25’x25’ square by the center point height of the square (see figure 8)
II. Accumulate values determined when multiplying area of a 0.7’x0.7’’ square by their center point height of the square (see figure 9)

13. Prior to calculating the volume of the dirt, predict the variability between the stockpile volumes calculated using the variety of survey points available to perform the measurement. Will the results be close to each other, or vary by 25% or more?
Students can estimate here - generally, the more squares we measure, the better our estimation should be. However, the students may expect that the measurements would still be close…

Data collected by UAS - Use this for questions 14 and 15…SfM actual pile volume is given in the excel document

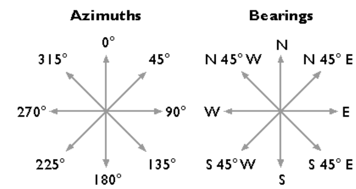
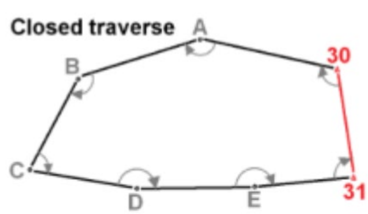
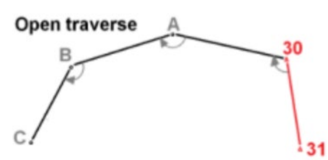
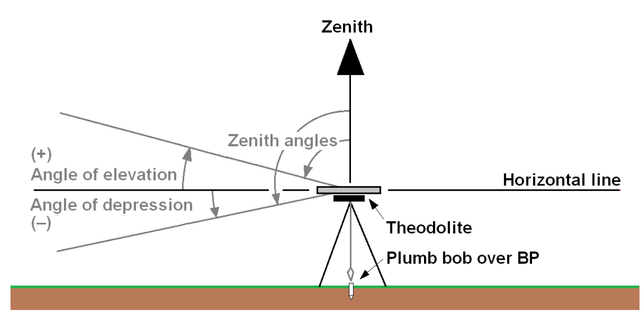
14. Method 1: Determine the stockpile volume in cubic feet (above 384 ft in elevation) using 39 survey points that were gathered every 25 ft across this dirt stock pile.
410,625 cubic feet

15. Method 2: Apply the formula derived to solve the first problem to the data points gathered every 0.7 ft, and compare this volume to Method 1, and the SfM (actual) pile volume.
418,646.40 cubic feet - this is closer to the actual volume than the 25 foot squares

Exit ticket – use the volume to calculate cost of removal of the dirt if each dump truck can haul 15 cubic yards of dirt at a cost of $120 per load.

$427142 \div 15$ = 28,477 loads at $120 per load = $3,417,136 to remove the dirt

Vocabulary List:

Angular Misclosure	The difference between the measured angles' sum on a traverse and the angle condition for the traverse configuration
Azimuth	horizontal angles that are measured from the reference meridian in the clockwise direction. used in compass surveying, plane surveying, where it is generally measured from the north. 
Misclosure	the situation where the last in a series of linked traverse lines fails to join up exactly with the first
Total station	Surveyors use a tool called a total station to measure dimensions – a variety of information is returned by the total station including lengths and angle measures  Figure 3: Total station operation (left), Back-site (right image shown on a yellow tripod), GPS shown on right image as black pole with tablet and round antenna
Ratio of precision	represents how many feet you would have to traverse with that accuracy to achieve 1 unit of error
Traverse	Traverse is a method in the field of surveying to establish control networks. It is also used in geodesy. Traverse networks involve placing survey stations along a line or path of travel, and then using the previously surveyed points as a base for observing the next point.  
Zenith Angle	the angle measured from directly above (the zenith) – so perfectly horizontal will have a zenith angle of 90°. If you are looking above horizontal,the zenith angle will be less than 90°. If you are looking below horizontal,the zenith angle will be greater than 90° 

Azimuth = Backsight Azimuth + Horizontal Angle

***side note: Azimuth measure between 0 and 360. If your calculated value for any Azimuth is above 360 or below 0, add or subtract 360 until you're back to an value between 0 and 360.*

***FS Azimuth = Front side azimuth (this is what Azimuth refers to in this example)*

1. Start with beginning BS Azimuth
2. Add Horizontal angle to next point to get your Azimuth
3. Add 180° to get your new BS Azimuth
4. Add Horizontal Angle to that BS Azimuth to get your next Azimuth
5. Repeat steps 3 and 4 as needed

Formulas:

$$\sum \text{latitudes} = \text{Misclosure in Northing Direction}$$

$$\sum \text{departures} = \text{Misclosure in Easting Direction}$$

$$\text{Linear Misclosure} = \sqrt{\left(\sum \text{latitudes}\right)^2 + \left(\sum \text{departures}\right)^2}$$

$$\text{Ratio of Precision} = \frac{\text{Total length of Traverse}}{\text{Linear Misclosure Value}}$$