Connecting Industry to Mathematics Instruction

NSF ATE Award # 1954291

Solutions: Key Up the Pressure - Water Storage Tank Student Activity Sheet

In partnership with WAKE TECH, WAKE COUNTY PUBLIC SCHOOL SYSTEM, WakeEd PARTNERSHIP

Overview:
When you turn on the faucet to get a drink of water or flush your toilet do you think about where that water comes from? The water that you use everyday is pumped through a network of underground pipes that connect consumers to a water treatment plant (WTP). While the water moves through this underground system at a constant rate, the demand for water by residential homes, businesses, and schools is constantly changing. Water storage tanks are located in the distribution system as a buffer between the supply and demand of potable water. Highfill is a civil engineering firm that specializes in water and wastewater infrastructure.

Task 1: Water Storage Tank Height and Water Pressure

Think about when you dive into a pool. The deeper you go the more pressure you feel in your ears.

Pressure in a water distribution system is typically between 40 and 80 PSI (PSI = pounds per square inch). Water pressure less than 40 PSI can cause problems with there not being enough water to take a shower or wash clothes. A water pressure greater than 80 PSI can cause damage to pipes.

1. What do you think that companies like Highfill need to consider when building a water storage tank?
   Possible answers:
   - Number of homes and businesses and schools that will be serviced by the tank.
   - The site of the tank and what type of soil it has.
   - Desired water pressure.

2. The height of water in the storage tank is directly proportional to pressure. If a water storage tank is serving a community that wants a desired water pressure of 65.6 PSI, guesstimate the height that the water tower would need to be. What things did you consider when making your guesstimate?
   This is purely a guess. Students may guess low amounts 15 ft to very high amounts like 500 ft. They will have the opportunity to review their guess later. Students may have considered the number of homes or businesses in the area or water pressure.

3. Another community has a water pressure of 50 PSI. How would the height of the water in the storage tank for this community compare to the height of the water in the storage tank for a community that has water pressure of 65.6 PSI?

Since the water pressure, 50 PSI is lower than the water pressure 65.6 PSI, the height of this water tank will be shorter than that of the water tank with 65.6 PSI

4. Water pressure is calculated using the following formula:

   $Pressure = height\ of\ water\ \times\ unit\ weight\ of\ water$. *Water at 70°F has a unit weight of 62.4 lbs/ft^3

   Suppose a water storage tank is filled to a point 140 feet above ground. Calculate the water pressure this water tank will produce in PSI (**pounds per square inch**)

   $$pressure = 140ft \times 62.4\frac{lbs}{ft^3} = 8736\frac{lbs}{ft^2}$$

   Convert from lbs/ft^2 to lbs/in^2

   $$8736\frac{lbs}{ft^2} \times \frac{1ft^2}{144in^2} = \frac{8736lbs}{144in^2} = 60.7\frac{lbs}{in^2} = 60.7\ PSI$$

5. Based on the PSI you calculated in #4, how would you adjust your guesstimate from # 2? Explain.
   Answers will vary based upon the student's guess. Students should explain that their guess was too high or too low.

Task 2: Weight and Soil Bearing Capacity:

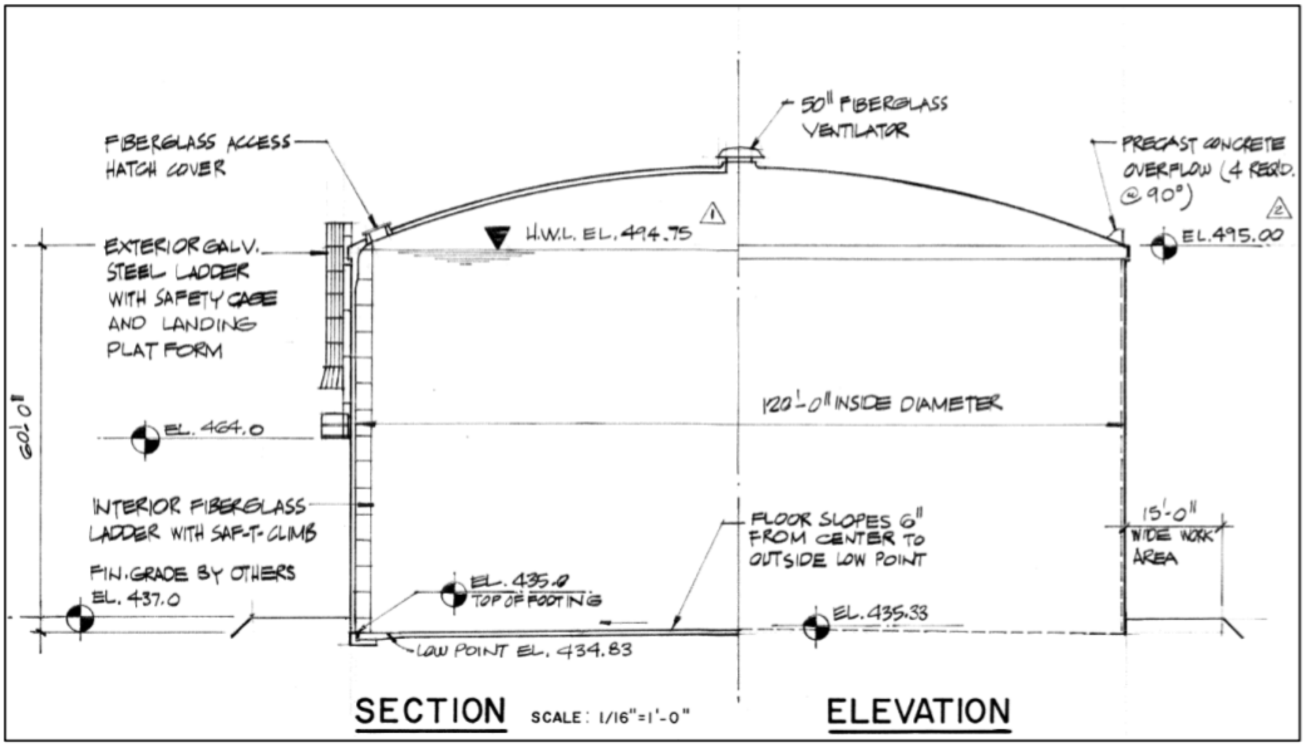
1. Civil engineers, like Highfill, must consider the weight of a storage tank (including the water) when designing a water storage tank. Why is this an important consideration?
   If the water tank is too heavy for the ground to support then the water tower may sink or collapse.

2. The weight of water in a storage tank can be calculated if you know the volume of the tank and its unit weight.

   $$Water\ Weight = Volume\ of\ water\ \times\ unit\ weight\ of\ water$$

   *Water at 70°F has a unit weight of 62.4 lbs/ft^3
   ** **The notation $\#/CF$ is commonly used by Highfill to represent** lbs/ft^3.

   Below is a drawing of a concrete ground storage tank with an inside diameter of 120 feet and a depth of 60 feet to the high-water line at the top of the cylindrical portion of the tank
   What is the weight of water when the tank is full? **Show your calculations.**



$$Weight\ of\ water = volume\ of\ water\ CF\ \times unit\ weight\ of\ water\ \#/CF$$

$$Volume = \pi r^2 \cdot H = (3.14)(60ft)^2(65ft) = 734{,}760\ ft^3$$

*Water at 70°F has a unit weight of 62.4 lbs/ft^3

$$Water\ Weight = 734{,}760\ ft^3 \cdot 62.4\ \frac{lbs}{ft^3} = 45{,}849{,}024\ lbs$$

3. In addition to the weight of the water, we must also consider the weight of the empty concrete tank. The tank wall is 15 inches thick and the floor is 4 inches thick. The top of the tank is fiberglass and weighs 170,000 #.

Inside radius of the tank = 60 ft, Thickness of wall = 15 in, Outside radius = 61.25 ft

$$Concrete\ Weight = Volume\ \ of\ concrete\ \times\ unit\ weight\ of\ concrete$$

*Reinforced concrete weighs $150\,\#/CF$.

a) What is the weight of the concrete tank?

Outside Volume of tank = π(61.25 ft)^2(60 ft) = 707,153 CF

Inside Volume of tank = π(60 ft)^2(60 ft) =678,584 CF

Volume of concrete tank = 707153 - 678584 = 28,569 CF

Volume of concrete floor = π(61.25 ft)^2(⅓ ft) = 3929 CF

Total volume of concrete = (28569 + 3929) = 32,498 CF

Concrete weight = 32,498 CF $\times$ 150 #/CF = 4,874,700 #

b) What is the weight of the concrete tank including the roof?

4,874,700 # + 170,000# = 5,044,700 #

c) What is the total weight of the tank when it is full?
Total weight of the tank when it is full = 45,849,024 # + 5,044,700 # = 50,893,724 #

4. Bearing pressure is the amount of pressure that a structure puts on the underlying soil in pounds per square feet, $\#/SF$.

$$Bearing\ Pressure\ = Total\ weight\ of\ concrete\ storage\ tank\ including\ the\ water \div Area\ of\ the\ tank\ floor$$

Calculate the bearing pressure of the tank above on the underlying soils.

Area of tank floor = π(61.25 ft)^2 = 11,786 SF

Bearing Pressure = 50,893,724 # ÷ 11,786 SF = 4318 lbs/ft^2

Geotechnical studies are typically performed on a storage tank site to determine the **bearing capacity** of the site. Bearing capacity is the maximum stress or pressure that the site can support without the failure of the soil or rock that is supporting the footing or base of the water tank.

They determine this by drilling and sampling the underlying soils, running laboratory tests, and running calculations. Some typical bearing capacity of soils are tabulated below:

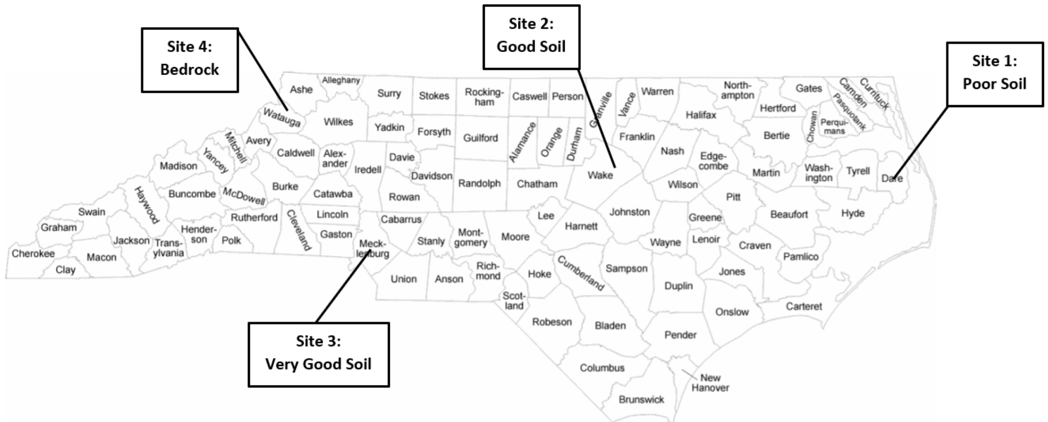
- Bedrock = 10,000 #/SF or more
- Very good soils = 5,000 #/SF
- Good soils = 3,000 #/SF
- Poor soils = 1,000 #/SF

If the bearing load of the tank is less than the capacity of the soils, the criteria is met. However, if the soils cannot support the bearing capacity, settlement may occur unless other alternatives are explored like increasing the area of the footing or installing a pile foundation).

5. What is the mathematical rationale for why increasing the area of the footing improves the soil's ability to support the water tank?

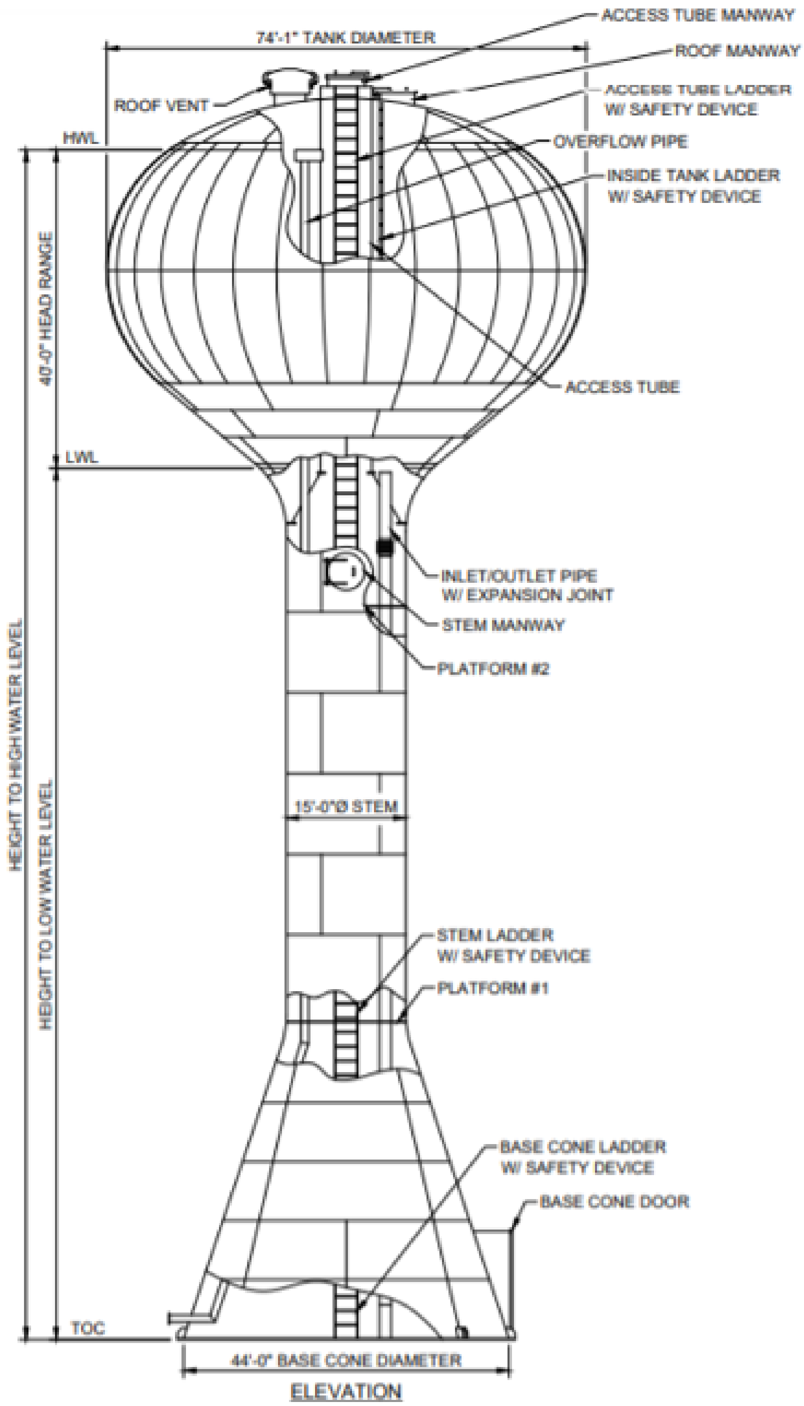
   Bearing pressure is found by dividing the weight of the storage tank by the area of the footing or base. If the area of the base is increased then the fraction, weight/area is decreased so the bearing pressure is less. (There is an inverse relationship between bearing pressure and area. As area increases, bearing pressure decreases.

6. Several sites needing a new water storage tank have been identified on the map below. On what site can the water tank in #4 be constructed and why?



Since the bearing pressure of the water storage tank is 4021 #/SF, the tank has to be built on a site that has a bearing capacity greater than 4021#/SF. Site 3 has very good soil and Site 4 has bedrock. The bearing capacity of very good soil is 5000 and bedrock has a bearing capacity of 10,000, therefore the water storage tank can be built on Sites 3 and 4.

7. A geotechnical study has determined that the site for the water storage tank below has very good soil. Will this site support the water storage tank, assuming the water tank is full? To simplify this task consider the tank to be a sphere with a diameter of 74ft 1in. The entire steel tank structure weighs 95,000 lbs. Prove your answer with calculations.



Diameter = 74 1/12 ft
Radius = 37 1/24 ft

Volume of Sphere = $4/3\ \pi\ r^3$
$= 4/3\pi(37\ 1/24)^3$
= 212,892 CF

Weight of water = 212,892 CF $\times$ 62.4 #/CF = 13,284,460 #

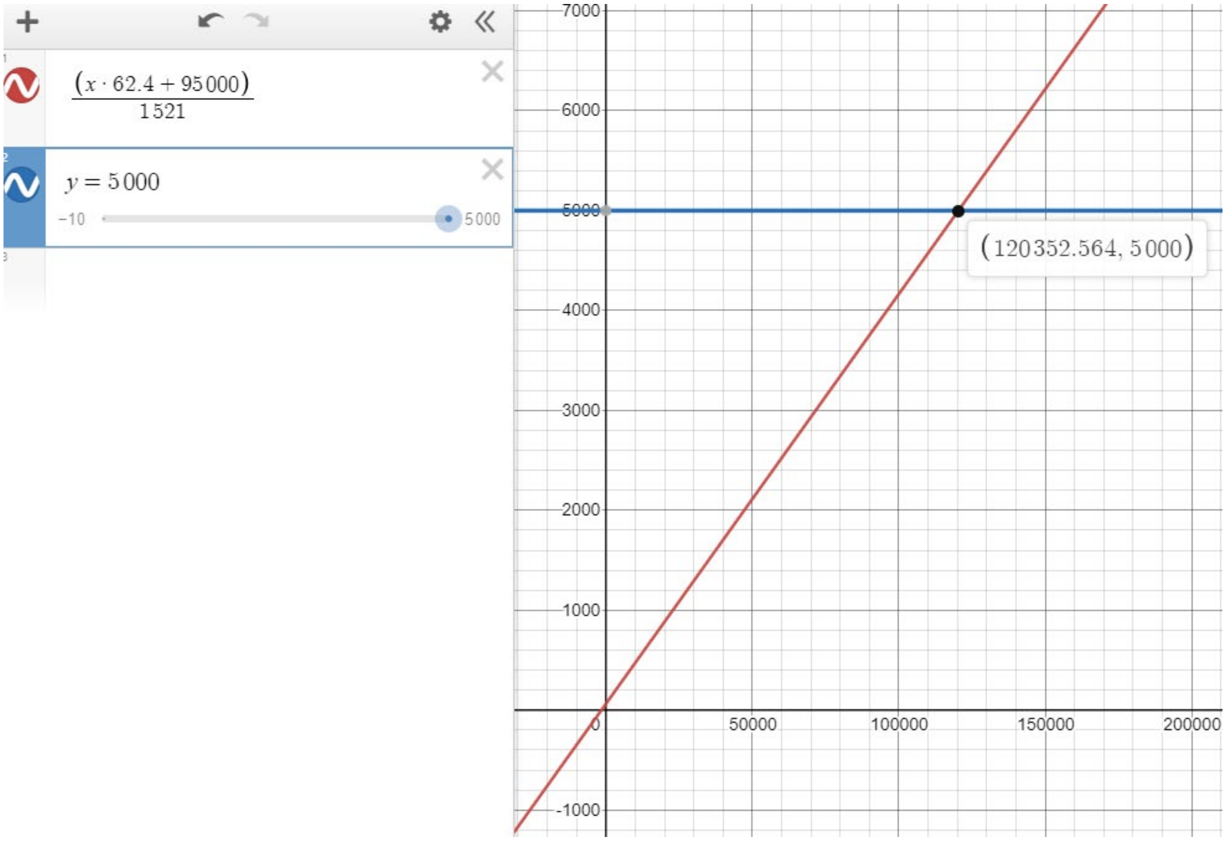
Total weight of full water tower = 13,284,461 + 95,000 = 13,379,461#

Area of base = $\pi(22)^2$= 1521 SF

Bearing Pressure = 13,379,461 ÷ 1521 = 8796 #/SF

This site has very good soil with a bearing capacity of 5000 so it will not support the water tower when filled.

8. What is the maximum amount (volume) of water that this spherical steel tank can hold given the site for the water storage tank has very good soil and the dimensions of the water tower remain unchanged?
This problem can be solved using a DESMOS Graph
Bearing Pressure =((Volume X 62.4) + 95000)
Bearing Capacity = 5000
Maximum Volume of the Tank is 120,353 CF


$\frac{(x \cdot 62.4 + 95000)}{1521}$
$y = 5000$
−10
5000
7000
6000
5000
4000
3000
2000
1000
0
-1000
50000
100000
150000
200000
(120352.564, 5000)