Connecting Industry to Mathematics Instruction
NSF ATE Award # 1954291

Solutions: Breathe Easy - Outside Air Requirements and Heating Load

Warm-ups

1. Determine the area of the rectangle below, in square feet. Round to 2 decimal places. 46.30 ft^2

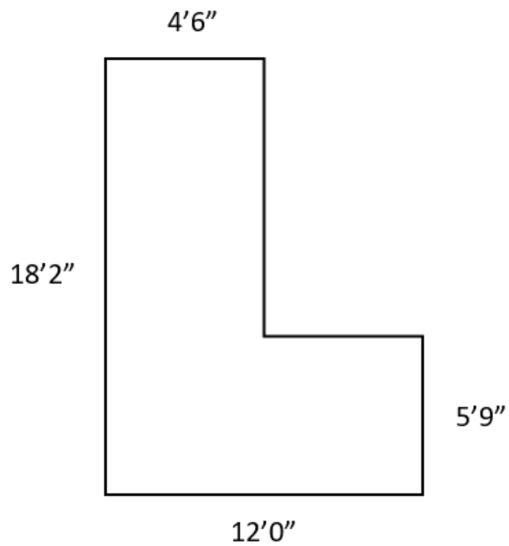
2. Determine the area of the rectangle above, in square meters. Round to 4 decimal places.

4.3014 m^2

3. Determine the two missing side lengths and find the area of the following figure:

   12’ 5” and 7’ 6”

$$124\frac{7}{8}ft^2$$



Task 1

Clean, fresh air plays a vital factor in the role of today's building designers.
How do technicians determine the amount of air that must flow into a certain space?

You are working to calculate the outside air requirements for the first floor of the provided floor plan. Using the provided information, determine the outside air requirements for each space.

Minimum Ventilation Rates Per NC Mechanical Code

	Occupant Density (Person/ $1000\ ft^2$)	People Outdoor Airflow Rate, R_p (CFM/Person)	Area Outdoor Airflow Rate, R_a(CFM/ft^2)
Office	6	5	0.06
Library	25	10	0.12
Conference room	50	5	0.06
Corridor	0	0	0.06
Lobby/Reception	30	5	0.06
Multi-function	100	7.5	0.06

Note: CFM = ft^3/min

Formula for required ventilation rate (airflow rate):

$$V = R_p P_z + R_a A_z,$$

where the variables/components of the formula are:

V =Ventilation rate in CFM

R_p =People outdoor airflow rate in CFM/person

P_z =Space occupancy, the whole # of people that the ventilation rate needs to account for

R_a =Area outdoor airflow rate in CFM/ft^2

A_z =Space floor area in ft^2

Exploration

1. Which two of the four components (variables) of the formula are provided in the table?
   R_p and R_a

2. Which two components will you need to determine?
   P_z and A_z

3. Which one of the two components which you must determine depends on the other?
   P_z depends on A_z so we will need to calculate A_z first, and then multiply by the occupant density

4. What are the units of the product $R_a A_z$?
   CFM

5. What are the units of the product $R_p P_z$?
   CFM

6. The occupant density for Lobby is listed as 30 people per 1000 sq. ft. Suppose the area of a lobby is 710 sq. ft.
   a. A spreadsheet set up by a coworker is calculating the space occupancy as 21300 people. What is wrong with the spreadsheet? Should be 21.3 people. The coworker forgot to divide by 1000.
   b. The space occupancy is the number of people which the ventilation system will have to account for when used in other calculations. Keeping this in mind, how should the correct space occupancy be rounded? Should round up to 22 people to be safe and account for all people.

Outside Air Requirements

	Space Floor Area A_z	Space Occupancy P_z	Ventilation Rate V
Offices (3 total)	120 x 3 = 360 ft^2	1 x 3 = 3 people	12.2 x 3 = 36.6 CFM
Library	1692.75 ft^2	43 people	633.13 CFM
Conference room	378 ft^2	19 people	117.68 CFM
Corridor	235 ft^2	0 people	14.1 CFM
Lobby/Reception*	818.3 ft^2	25 people	174.1 CFM
Multi-function	378 ft^2	38 people	307.68 CFM

*Include hallway between toilets and elevators and stairway in Lobby/Reception

Example: Office space

Step 1: Find the area of the space.

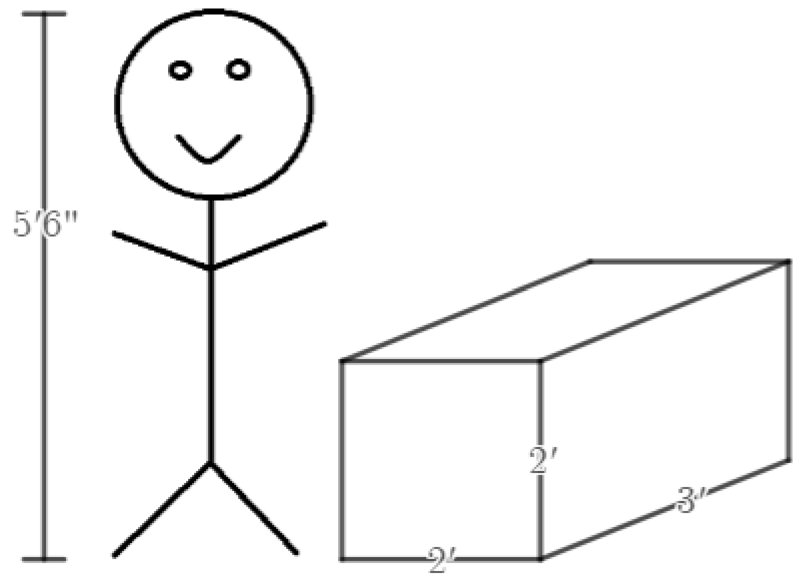
Step 2: Determine the amount of people (the space occupancy P_z), using the surface area and the occupant density. (Round up)

Step 3: Use the table and the formula to find the Ventilation Rate.

1. Area of each office: $12\ x\ 10\ =\ 120\ ft^2$
2. Amount of people (P_z): $\dfrac{6\ people}{1000\ ft^2} = \dfrac{x}{120\ ft^2}$; $x = 0.72\ people$; rounds up to $P_z = 1$ person
3. Ventilation Rate, V: $V = R_p P_z + R_a A_z = (5\ CFM/person) \times\ (1\ person)\ +$ $(0.06\ CFM/ft^2) \times (\ 120\ ft^2)$

$$V = 5\ CFM\ +\ 7.2\ CFM\ =\ 12.2\ CFM$$

So, each office requires 12.2 CFM of ventilation. This is approximately equivalent to a 2ft x 2ft x 3ft box of air being forced into the room each minute.



Exercise

Complete the Outside Air Requirements Table for the remaining spaces: library, conference, corridor, multi-function, lobby/reception. Compare the outside air requirements for each space and discuss the results with your group.

Task 2

Keeping a building warm in the winter and cool in the summer is a main concern for architectural design companies.

For our scenario, you are working to keep the building occupants warm in the winter by calculating the heating load for the exterior walls and windows of the rooms on the first floor. Which component contributes more to the heating load? The windows because more heat is transferred through these than walls.

Formula for heating or cooling load:

$$Q = UA\,(T_i - T_o),$$

where the variables/components of the formula are:

Q =Heating or cooling load in BTU/hr (amount of energy needed to keep space heated)

U =Total U-value for assembly in $BTU/(hr \times ft^2 \times \ {}^\circ F)$

A =Area in ft^2

T_i = Indoor temperature in F

T_o = Outdoor temperature in F

The temperatures for Raleigh, NC, are given below.

	Winter	Summer
Peak Outdoor Design Conditions for Raleigh, NC, T_o	$20.4^\circ F$	$95^\circ F$
Indoor Design Conditions for Building, T_i	$70^\circ F$	$75^\circ F$

The exterior components of the building are insulated to prevent heat exchange. Each component is assigned a U-Value, which has units $BTU/(hr \times ft^2 \times \ {}^\circ F)$. The unit BTU is a measure of energy (1000 BTU is approximately 0.293071 kWh) called a British thermal unit.

	U-Value in $BTU/(hr \times ft^2 \times \ {}^\circ F)$
Wall	0.064
Window	0.4
Roof	0.048

For this floor plan, the exterior wall height is $13'$, and a typical window is $3' \times 4'$.

Exploration

1. Why should we only include exterior walls in the calculation for heating load?
   Because these are the parts of the rooms that are responsible for the most heat loss/cooling loss in relation to the outdoors.

2. Which three of the four components (variables) of the formula are provided in the information above?
   U-value, T_i and T_o
3. Which component will you need to determine?
   A (Area of the walls/windows)

Example (Corner Office)

Determine the heating load caused by the corner office. Because the U-values are given by wall/window, we will need to do each separately.

Step 1: Find the area of the windows.

$3' \times 4' = 12\ ft^2$; Total of $2 \times 12 = 24\ ft^2$

Step 2: Calculate the Heating Load caused by the windows, using the formula above.

$$Q = UA\,(T_i - T_o) = (0.4 \frac{BTU}{hr \times ft^2 \times \circ F})(24\ ft^2)(70 \quad {}^\circ F - 20.4 \quad {}^\circ F) = 476.16\ BTU/hr$$

Step 3: Find the area of the exterior walls of the corner office (without the windows).

$12' + 10' = 22'$ length of exterior wall, at a height of $13'$ gives $22' \times 13' = 286\ ft^2$

subtracting the windows gives $286 - 24 = 262\ ft^2$

Step 4: Calculate the Heating Load using the formula above.

$$Q = UA\,(T_i - T_o) = (0.064 \frac{BTU}{hr \times ft^2 \times \circ F})(262\ ft^2)(70 \quad {}^\circ F - 20.4 \quad {}^\circ F) =$$

$$831.69\ BTU/hr$$

Step 5: Add the heating loads from the windows and the walls:

Total $Q = 476.16\ BTU/hr + 838.04\ BTU/hr = 1314.2\ BTU/hr$

Corner Office	Area (ft^2)	Heating Load (BTU/hr)
Windows	24	476.16
Walls	262	831.69

So, the walls contribute more to the heating load.

Exercises

1. Determine the heating load for the conference room. Only consider external walls/windows.

Conference Room	Area (ft^2)	Heating Load (BTU/hr)
Windows	24	476.16
Walls	210	666.62

2. Determine the heating load for the library. Only consider external walls/windows; you may treat the double door onto the patio as a wall.

Library	Area (ft^2)	Heating Load (BTU/hr)
Windows	292	5793.28
Walls	1217.67	3865.37

Discussion

3. For each room above (corner office, conference room, library) determine the component which contributes the most to the heating load.

   Office: Walls more

   Conference Room: Walls more (but slightly closer)

   Library: Windows more

4. Explain why all three rooms did not have the same component that contributed the most to the heating load. The library had so many windows and the U-value for windows is much greater than that of the walls.

5. Why is it not enough to just compare the area of windows to the area of walls? The U-values are very different; windows leak air and heat at a much greater rate/quantity than walls.

6. Consider the formula for the heating load $Q = UA\,(T_i - T_o)$.
   a. What will happen to the heating load if the height of the wall is decreased? The heating load will decrease.
   b. Suppose another wall assembly is chosen with "better" insulation. Should this correspond to a greater or smaller U-value? Smaller U-value
   c. Which month(s) of the year will the heating/cooling bills be most expensive? How does this relate to the formula? December – February and June – August. The outdoor temperatures are further away from the indoor temperatures during these months which causes the heating load, Q to increase.